International Baccalaureate®
Baccalauréat International
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MARKSCHEME

May 2008

MATHEMATICS

Higher Level

Paper 2

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

  Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1****(AP)****. Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) Use of $\bar{x} = \frac{\sum_{i=1}^{4} x_i}{n}$ *(M1)*

$\bar{x} = \frac{(k-2) + k + (k+1) + (k+4)}{4}$ *(A1)*

$\bar{x} = \frac{4k+3}{4} \quad \left(= k + \frac{3}{4}\right)$ ***A1*** ***N3***

(b) Either attempting to find the new mean or subtracting 3 from **their** $\bar{x}$ *(M1)*

$\bar{x} = \frac{4k+3}{4} - 3 \qquad \left(= \frac{4k-9}{4},\ k - \frac{9}{4}\right)$ ***A1*** ***N2***

[5 marks]

2. (a) Either finding depths graphically, using $\sin\frac{\pi t}{6} = \pm 1$ or solving $h'(t) = 0$ for t *(M1)*

$h(t)_{\text{max}} = 12 \text{ (m)},\ h(t)_{\text{min}} = 4 \text{ (m)}$ ***A1A1*** ***N3***

(b) Attempting to solve $8 + 4\sin\frac{\pi t}{6} = 8$ algebraically or graphically *(M1)*

$t \in [0, 6] \cup [12, 18] \cup \{24\}$ ***A1A1*** ***N3***

[6 marks]

3. (a) Either solving $e^{-x} - x + 1 = 0$ for x, stating $e^{-x} - x + 1 = 0$, stating $P(x, 0)$ or using an appropriate sketch graph. ***M1***

$x = 1.28$ ***A1*** ***N1***

Note: Accept $P(1.28, 0)$.

(b) Area $= \int_0^{1.278...} (e^{-x} - x + 1)\, dx$ ***M1A1***

$= 1.18$ ***A1*** ***N1***

Note: Award ***M1A0A1*** if the dx is absent.

[5 marks]

4. Attempting to find the mode graphically or by using $f'(x) = 12x(2-3x)$ ***(M1)***

Mode $= \dfrac{2}{3}$ ***A1***

Use of $\mathrm{E}(X) = \int_0^1 x\,f(x)\,\mathrm{d}x$ ***(M1)***

$\mathrm{E}(X) = \dfrac{3}{5}$ ***A1***

$\int_{\frac{3}{5}}^{\frac{2}{3}} f(x)\,\mathrm{d}x = 0.117 \quad \left(= \dfrac{1981}{16\,875}\right)$ ***M1A1*** ***N4***

[6 marks]

5. **METHOD 1**

Attempting to use the cosine rule *i.e.* $\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2 - 2 \times \mathrm{AB} \times \mathrm{AC} \times \cos \mathrm{B\hat{A}C}$ ***(M1)***

$6^2 = 8.75^2 + \mathrm{AC}^2 - 2 \times 8.75 \times \mathrm{AC} \times \cos 37.8^\circ$ (or equivalent) ***A1***

Attempting to solve the quadratic in AC *e.g.* graphically, numerically or with quadratic formula ***M1A1***

Evidence from a sketch graph or their quadratic formula ($\mathrm{AC} = \ldots$) that there are two values of AC to determine. ***(A1)***

$\mathrm{AC} = 9.60$ or $\mathrm{AC} = 4.22$ ***A1A1*** ***N4***

Note: Award ***(M1)A1M1A1(A0)A1A0*** for one correct value of AC.

[7 marks]

METHOD 2

Attempting to use the sine rule *i.e.* $\dfrac{\mathrm{BC}}{\sin \mathrm{B\hat{A}C}} = \dfrac{\mathrm{AB}}{\sin \mathrm{A\hat{C}B}}$ ***(M1)***

$\sin C = \dfrac{8.75 \sin 37.8^\circ}{6} \quad (= 0.8938...)$ ***(A1)***

$C = 63.3576...^\circ$ ***A1***

$C = 116.6423...^\circ$ **and** $B = 78.842...^\circ$ or $B = 25.5576...^\circ$ ***A1***

EITHER

Attempting to solve $\dfrac{\mathrm{AC}}{\sin 78.842...^\circ} = \dfrac{6}{\sin 37.8^\circ}$ or $\dfrac{\mathrm{AC}}{\sin 25.5576...^\circ} = \dfrac{6}{\sin 37.8^\circ}$ ***M1***

OR

Attempting to solve $\mathrm{AC}^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 25.5576...^\circ$ or

$\mathrm{AC}^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 78.842...^\circ$ ***M1***

$\mathrm{AC} = 9.60$ or $\mathrm{AC} = 4.22$ ***A1A1*** ***N4***

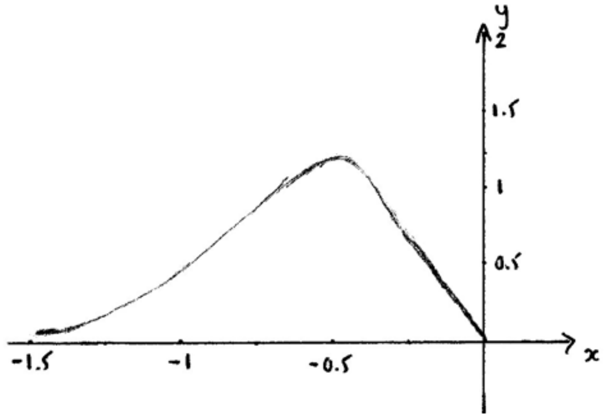
Note: Award ***(M1)(A1)A1A0M1A1A0*** for one correct value of AC.

[7 marks]

6. **METHOD 1**

EITHER

Using the graph of $y = f'(x)$ ***(M1)***

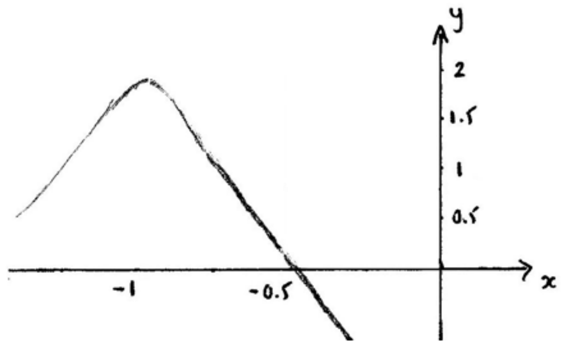


A1

The maximum of $f'(x)$ occurs at $x = -0.5$. ***A1***

OR

Using the graph of $y = f''(x)$. ***(M1)***



A1

The zero of $f''(x)$ occurs at $x = -0.5$. ***A1***

THEN

Note: Do not award this ***A1*** for stating $x = \pm 0.5$ as the final answer for x.

$f(-0.5) = 0.607\ (= e^{-0.5})$ ***A2***

Note: Do not award this ***A1*** for also stating $(0.5, 0.607)$ as a coordinate.

continued ...

Question 6 continued

EITHER

Correctly labelled graph of $f'(x)$ for $x<0$ denoting the maximum $f'(x)$ ***R1***
(*e.g.* $f'(-0.6)=1.17$ and $f'(-0.4)=1.16$ stated) ***A1*** ***N2***

OR

Correctly labelled graph of $f''(x)$ for $x<0$ denoting the maximum $f'(x)$ ***R1***
(*e.g.* $f''(-0.6)=0.857$ and $f''(-0.4)=-1.05$ stated) ***A1*** ***N2***

OR

$f'(0.5)\approx 1.21$. $f'(x)<1.21$ just to the left of $x=-\frac{1}{2}$
and $f'(x)<1.21$ just to the right of $x=-\frac{1}{2}$ ***R1***
(*e.g.* $f'(-0.6)=1.17$ and $f'(-0.4)=1.16$ stated) ***A1*** ***N2***

OR

$f''(x)>0$ just to the left of $x=-\frac{1}{2}$ and $f''(x)<0$ just to the right of $x=-\frac{1}{2}$ ***R1***
(*e.g.* $f''(-0.6)=0.857$ and $f''(-0.4)=-1.05$ stated) ***A1*** ***N2***

[7 marks]

continued ...

Question 6 continued

METHOD 2

$f'(x) = -4x\mathrm{e}^{-2x^2}$ ***A1***

$f''(x) = -4\mathrm{e}^{-2x^2} + 16x^2\mathrm{e}^{-2x^2} \quad \left(= (16x^2 - 4)\mathrm{e}^{-2x^2}\right)$ ***A1***

Attempting to solve $f''(x) = 0$ ***(M1)***

$x = -\frac{1}{2}$ ***A1***

Note: Do not award this ***A1*** for stating $x = \pm\frac{1}{2}$ as the final answer for x.

$f\left(-\frac{1}{2}\right) = \frac{1}{\sqrt{\mathrm{e}}} \quad (= 0.607)$ ***A1***

Note: Do not award this ***A1*** for also stating $\left(\frac{1}{2}, \frac{1}{\sqrt{\mathrm{e}}}\right)$ as a coordinate.

EITHER

Correctly labelled graph of $f'(x)$ for $x < 0$ denoting the maximum $f'(x)$ ***R1***

(*e.g.* $f'(-0.6) = 1.17$ and $f'(-0.4) = 1.16$ stated) ***A1*** ***N2***

OR

Correctly labelled graph of $f''(x)$ for $x < 0$ denoting the maximum $f'(x)$ ***R1***

(*e.g.* $f''(-0.6) = 0.857$ and $f''(-0.4) = -1.05$ stated) ***A1*** ***N2***

OR

$f'(0.5) \approx 1.21$. $f'(x) < 1.21$ just to the left of $x = -\frac{1}{2}$

and $f'(x) < 1.21$ just to the right of $x = -\frac{1}{2}$ ***R1***

(*e.g.* $f'(-0.6) = 1.17$ and $f'(-0.4) = 1.16$ stated) ***A1*** ***N2***

OR

$f''(x) > 0$ just to the left of $x = -\frac{1}{2}$ and $f''(x) < 0$ just to the right of $x = -\frac{1}{2}$ ***R1***

(*e.g.* $f''(-0.6) = 0.857$ and $f''(-0.4) = -1.05$ stated) ***A1*** ***N2***

[7 marks]

7. (a) $X \sim \mathrm{B}(n, 0.4)$ ***(A1)***

Using $\mathrm{P}(X = x) = \binom{n}{r}(0.4)^x (0.6)^{n-x}$ ***(M1)***

$\mathrm{P}(X = 2) = \binom{n}{2}(0.4)^2 (0.6)^{n-2} \quad \left(= \frac{n(n-1)}{2}(0.4)^2(0.6)^{n-2}\right)$ ***A1*** ***N3***

(b) $\mathrm{P}(X = 2) = 0.121$ ***A1***

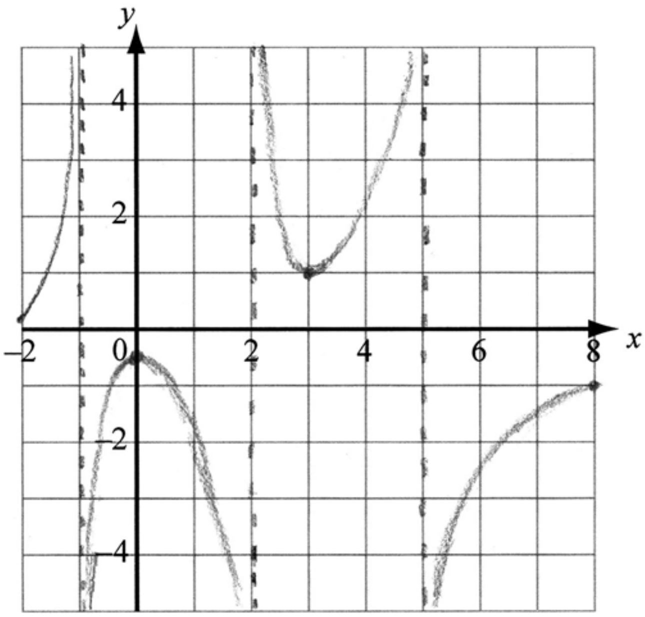
Using an appropriate method (including trial and error) to solve their equation. ***(M1)***

$n = 10$ ***A1*** ***N2***

Note: Do not award the last ***A1*** if any other solution is given in their final answer.

[6 marks]

8.



A1A1A1A1A1

Notes: Award ***A1*** for vertical asymptotes at $x = -1$, $x = 2$ and $x = 5$.

A1 for $x \to -2$, $\frac{1}{f(x)} \to 0^+$

A1 for $x \to 8$, $\frac{1}{f(x)} \to -1$

A1 for local maximum at $\left(0, -\frac{1}{2}\right)$ (branch containing local max. must be present)

A1 for local minimum at $(3, 1)$ (branch containing local min. must be present)

In each branch, correct asymptotic behaviour must be displayed to obtain the ***A1***.

Disregard any stated horizontal asymptotes such as $y = 0$ or $y = -1$.

[5 marks]

9. **METHOD 1**

Substituting $z = x + \mathrm{i}y$ to obtain $w = \dfrac{x + y\mathrm{i}}{(x + y\mathrm{i})^2 + 1}$ ***(A1)***

$w = \dfrac{x + y\mathrm{i}}{x^2 - y^2 + 1 + 2xy\mathrm{i}}$ ***A1***

Use of $(x^2 - y^2 + 1 - 2xy\mathrm{i})$ to make the denominator real. ***M1***

$= \dfrac{(x + y\mathrm{i})(x^2 - y^2 + 1 - 2xy\mathrm{i})}{(x^2 - y^2 + 1)^2 + 4x^2y^2}$ ***A1***

$\operatorname{Im} w = \dfrac{y(x^2 - y^2 + 1) - 2x^2y}{(x^2 - y^2 + 1)^2 + 4x^2y^2}$ ***(A1)***

$= \dfrac{y(1 - x^2 - y^2)}{(x^2 - y^2 + 1)^2 + 4x^2y^2}$ ***A1***

$\operatorname{Im} w = 0 \quad \Rightarrow 1 - x^2 - y^2 = 0$ *i.e.* $|z| = 1$ as $y \neq 0$ ***R1AG*** ***N0***

[7 marks]

METHOD 2

$w(z^2 + 1) = z$ ***(A1)***

$w(x^2 - y^2 + 1 + 2\mathrm{i}xy) = x + y\mathrm{i}$ ***A1***

Equating real and imaginary parts

$w(x^2 - y^2 + 1) = x$ and $2wx = 1,\ y \neq 0$ ***M1A1***

Substituting $w = \dfrac{1}{2x}$ to give $\dfrac{x}{2} - \dfrac{y^2}{2x} + \dfrac{1}{2x} = x$ ***A1***

$-\dfrac{1}{2x}(y^2 - 1) = \dfrac{x}{2}$ or equivalent ***(A1)***

$x^2 + y^2 = 1$, *i.e.* $|z| = 1$ as $y \neq 0$ ***R1AG***

[7 marks]

10. Attempting to solve $\left|0.1x^2 - 2x + 3\right| = \log_{10} x$ numerically or graphically. ***(M1)***

$x = 1.52,\ 1.79$ ***(A1)(A1)***

$x = 17.6,\ 19.1$ ***(A1)***

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$ ***A1A1*** ***N2***

[6 marks]

SECTION B

11. (a) (i) $P(4.8 < X < 7.5) = P(-0.8 < Z < 1)$ ***(M1)***
$= 0.629$ ***A1*** ***N2***

Note: Accept $P(4.8 \le X \le 7.5) = P(-0.8 \le Z \le 1)$.

(ii) Stating $P(X < d) = 0.15$ **or** sketching an appropriately labelled diagram. ***A1***
$\frac{d-6}{1.5} = -1.0364...$ ***(M1)(A1)***
$d = (-1.0364...)(1.5) + 6$ ***(M1)***
$= 4.45$ (km) ***A1*** ***N4***
[7 marks]

(b) Stating **both** $P(X > 8) = 0.1$ and $P(X < 2) = 0.05$ **or** sketching an appropriately labelled diagram. ***R1***
Setting up two equations in μ and σ ***(M1)***
$8 = \mu + (1.281...)\sigma$ **and** $2 = \mu - (1.644...)\sigma$ ***A1***
Attempting to solve for μ and σ (including by graphical means) ***(M1)***
$\sigma = 2.05$ (km) **and** $\mu = 5.37$ (km) ***A1A1*** ***N4***

Note: Accept $\mu = 5.36, 5.38$.

[6 marks]

(c) (i) Use of the Poisson distribution in an inequality. ***M1***
$P(T \ge 3) = 1 - P(T \le 2)$ ***(A1)***
$= 0.679...$ ***A1***
Required probability is $(0.679...)^2 = 0.461$ ***M1A1*** ***N3***

Note: Allow ***FT*** for their value of $P(T \ge 3)$.

(ii) $\tau \sim Po(17.5)$ ***A1***
$P(\tau = 15) = \frac{e^{-17.5}(17.5)^{15}}{15!}$ ***(M1)***
$= 0.0849$ ***A1*** ***N2***
[8 marks]

Total [21 marks]

12. (a) (i) Attempting to find $\boldsymbol{M}^2$ ***M1***

$\boldsymbol{M}^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$ ***A1***

$b(a+d)=b$ or $c(a+d)=c$ ***A1***

Hence $a+d=1$ (as $b \neq 0$ or $c \neq 0$) ***AG*** ***N0***

(ii) $a^2+bc=a$ ***M1***

$\Rightarrow bc = a - a^2 \ \left(= a(1-a)\right)$ ***A1*** ***N1***

[5 marks]

(b) **METHOD 1**

Using $\det \boldsymbol{M} = ad - bc$ ***M1***

$\det \boldsymbol{M} = ad - a(1-a)$ or $\det \boldsymbol{M} = a(1-a) - a(1-a)$ (or equivalent) ***A1***

$= 0$ using $a+d=1$ or $d = 1-a$ to simplify their expression ***R1***

Hence $\boldsymbol{M}$ is a singular matrix ***AG*** ***N0***

[3 marks]

METHOD 2

Using $bc = a(1-a)$ and $a+d=1$ to obtain $bc = ad$ ***M1A1***

$\det \boldsymbol{M} = ad - bc$ and $ad - bc = 0$ as $bc = ad$ ***R1***

Hence $\boldsymbol{M}$ is a singular matrix ***AG*** ***N0***

[3 marks]

(c) $a(1-a) > 0$ ***(M1)***

$0 < a < 1$ ***A1A1*** ***N3***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequality signs.

[3 marks]

continued ...

Question 12 continued

(d) **METHOD 1**

Attempting to expand $(\boldsymbol{I}-\boldsymbol{M})^2$ ***M1***

$(\boldsymbol{I}-\boldsymbol{M})^2 = \boldsymbol{I}-2\boldsymbol{M}+\boldsymbol{M}^2$ ***A1***

$= \boldsymbol{I}-2\boldsymbol{M}+\boldsymbol{M}$ ***A1***

$= \boldsymbol{I}-\boldsymbol{M}$ ***AG*** ***N0***

[3 marks]

METHOD 2

Attempting to expand $(\boldsymbol{I}-\boldsymbol{M})^2 = \begin{pmatrix} 1-a & -b \\ -c & 1-d \end{pmatrix}^2$ (or equivalent) ***M1***

$(\boldsymbol{I}-\boldsymbol{M})^2 = \begin{pmatrix} (1-a)^2+bc & -b(1-a)-b(1-d) \\ -c(1-a)-c(1-d) & bc+(1-d)^2 \end{pmatrix}$ (or equivalent) ***A1***

Use of $a+d=1$ and $bc=a-a^2$ to show desired result. ***M1***

Hence $(\boldsymbol{I}-\boldsymbol{M})^2 = \begin{pmatrix} 1-a & -b \\ -c & 1-d \end{pmatrix}$ ***AG*** ***N0***

[3 marks]

(e) (Let $P(n)$ be $(\boldsymbol{I}-\boldsymbol{M})^n = \boldsymbol{I}-\boldsymbol{M}$)

For $n=1$: $(\boldsymbol{I}-\boldsymbol{M})^1 = \boldsymbol{I}-\boldsymbol{M}$, so $P(1)$ is true ***A1***

Assume $P(k)$ is true, *i.e.* $(\boldsymbol{I}-\boldsymbol{M})^k = \boldsymbol{I}-\boldsymbol{M}$ ***M1***

Consider $P(k+1)$

$(\boldsymbol{I}-\boldsymbol{M})^{k+1} = (\boldsymbol{I}-\boldsymbol{M})^k(\boldsymbol{I}-\boldsymbol{M})$ ***M1***

$= (\boldsymbol{I}-\boldsymbol{M})(\boldsymbol{I}-\boldsymbol{M}) \quad \left(= (\boldsymbol{I}-\boldsymbol{M})^2\right)$ ***A1***

$= (\boldsymbol{I}-\boldsymbol{M})$ ***A1***

$P(k)$ true implies $P(k+1)$ true, $P(1)$ true so $P(n)$ true $\forall n \in \mathbb{Z}^+$ ***R1*** ***N0***

[6 marks]

Total [20 marks]

13. (a) (i) **EITHER**

Attempting to separate the variables *(M1)*

$$\frac{\mathrm{d}v}{-v(1+v^2)} = \frac{\mathrm{d}t}{50}$$ *(A1)*

OR

Inverting to obtain $\frac{\mathrm{d}t}{\mathrm{d}v}$ *(M1)*

$$\frac{\mathrm{d}t}{\mathrm{d}v} = \frac{-50}{v(1+v^2)}$$ *(A1)*

THEN

$$t = -50\int_{10}^{5} \frac{1}{v(1+v^2)}\mathrm{d}v \quad \left(= 50\int_{5}^{10} \frac{1}{v(1+v^2)}\mathrm{d}v \right)$$ ***A1*** ***N3***

(ii) $t = 0.732 \text{ (sec)} \quad \left(= 25\ln\frac{104}{101} \text{(sec)} \right)$ ***A2*** ***N2***

[5 marks]

continued ...

Question 13 continued

(b) (i) $\frac{dv}{dt} = v\frac{dv}{dx}$ ***(M1)***

Must see division by v ($v > 0$) ***A1***

$\frac{dv}{dx} = \frac{-(1+v^2)}{50}$ ***AG*** ***N0***

(ii) Either attempting to separate variables or inverting to obtain $\frac{dx}{dv}$ ***(M1)***

$\int \frac{dv}{1+v^2} = -\frac{1}{50}\int dx$ (or equivalent) ***A1***

Attempting to integrate both sides ***M1***

$\arctan v = -\frac{x}{50} + C$ ***A1A1***

Note: Award ***A1*** for a correct LHS and ***A1*** for a correct RHS that must include C.

When $x = 0$, $v = 10$ and so $C = \arctan 10$ ***M1***

$x = 50(\arctan 10 - \arctan v)$ ***A1*** ***N1***

(iii) Attempting to make $\arctan v$ the subject. ***M1***

$\arctan v = \arctan 10 - \frac{x}{50}$ ***A1***

$v = \tan\left(\arctan 10 - \frac{x}{50}\right)$ ***M1A1***

Using $\tan(A - B)$ formula to obtain the desired form. ***M1***

$v = \frac{10 - \tan\frac{x}{50}}{1 + 10\tan\frac{x}{50}}$ ***AG*** ***N0***

[14 marks]

Total [19 marks]